ÉCOLE GLOBALE
INTERNATIONAL GIRLS' SCHOOL
Dehradun

HOLIDAY HOMEWORK **SUMMER BREAK 2018-19**

CLASS XII CBSE **SUBJECT : MATHEMATICS**

S No.	Questions	Marks
1	Evaluate : $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$	1
2	. Solve for x : $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x;\ x>0$	1
3	What is the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?	1
4	What is the domain of the function $\sin^{-1}x$?	1
5	What is the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?	1
6	. Write the value of $\tan\left(2\tan^{-1}\frac{1}{5}\right)$.	1
7	Write the value of $\sin\left(2\sin^{-1}\frac{3}{5}\right)$.	1
8	Prove the following: $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$	4
9	Prove that $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$.	4
10	Prove that: $\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$ ***OR*** Solve for x : $\tan^{-1}3x+\tan^{-1}2x=\frac{\pi}{4}$	4
11	. Prove the following: $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)-\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$.	4

12	Solve the following for x: $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)+\tan^{-1}\left(\frac{2x}{x^2-1}\right)=\frac{2\pi}{3}$.	4
13	Solve for x: $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x), x\neq\frac{\pi}{2}$	4
14	Prove the following : $\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ **OR** Prove the following: $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]=\sqrt{\frac{1+x^2}{2+x^2}}$	4
15	Show that: $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$ **OR** Solve the following equation: $\cos(\tan^{-1}x)=\sin\left(\cot^{-1}\frac{3}{4}\right)$	4
14	If $y=\cot^{-1}(\sqrt{\cos x})-\tan^{-1}(\sqrt{\cos x})$, then prove that $\sin y=\tan^2\left(\frac{x}{2}\right)$.	4
15	Prove that $2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$.	4
16	Solve for x : $\cos(\tan^{-1}x)=\sin\left(\cot^{-1}\frac{3}{4}\right)$ **OR** Prove that : $\cos^{-1}7+\cot^{-1}8+\cot^{-1}18=\cot^{-1}3$	4
17	Prove the following: $\cos\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)=\frac{6}{5\sqrt{13}}$	4
18	Prove that : $\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)=\pi$.	4
19	. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$. Find the cofactor of a_{12} in the following: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$	1
20	For a 2 × 2 matrix, $A=[a_{ij}]$, whose elements are given by $a_{ij}=\frac{i}{j}$, write the value of a_{12}. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular? Write A^{-1} for $A=\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$	1

21	Evaluate: $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$ If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, write A^{-1} in terms of A. If a matrix has 5 elements, write all possible orders it can have.	1
22	If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23}. If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, write the value of x.	1
23	For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix? If matrix $A - \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 - kA$, then write the value of k.	1
24	If A is a square matrix and $\|A\| = 2$, then write the value of $\|AA'\|$, where A' is the transpose of matrix A.	1
25	Use elementary column operation $C_2 \to C_2 - 2C_1$ in the matrix equation $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$. If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$ write the value of $a - 2b$. If A is a 3×3 matrix, $\|A\| \neq 0$ and $\|3A\| = k\,\|A\|$, then write the value of k.	1
26	Using properties of determinants, prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$	4
27	Using properties of determinants, prove that $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$	4
28	Using properties of determinants, solve the following for x: $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} - 0$	4

29	Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as sum of two matrices such that one is symmetric and the other is skew symmetric. ***OR*** If $A = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$, verify that $A^2 - 4A - 5I = 0$	4
30	Using properties of determinants, prove that $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$	4
31	Using properties of determinants, prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$	4
32	Prove the following, using properties of determinants : $\begin{vmatrix} a+bx^2 & c+dx^2 & p+qx^2 \\ ax^2+b & cx^2+d & px^2+q \\ u & v & w \end{vmatrix} = (x^4-1)\begin{vmatrix} b & d & q \\ a & c & p \\ u & v & w \end{vmatrix}$ ***OR*** Using elementary transformations, find the inverse of the following matrix : $A = \begin{pmatrix} 6 & 5 \\ 5 & 4 \end{pmatrix}$.	4
33	. Using properties of determinants, prove that $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$	4
34	Prove the following, using properties of determinants : $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2(3abc - a^3 - b^3 - c^3)$	4

35	Using properties of determinants, prove the following : $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$ $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$	6
36	. Using matrices, solve the following system of linear equations: $2x - y + z = 3$ $-x + 2y - z = -4$ $x - y + 2z = 1$ ***OR*** Using elementary transformations, find the inverse of the following matrix: $\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$	6
37	Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations : $x - y + 2z = 1$ $2y - 3z = 1$ $3x - 2y + 4z = 2$	6
38	If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.	6
39	*The management committee of a residential colony decided to award some of members (say x) for honesty, some (say y) for helping others and some others (say z) supervising the workers to keep the colony neat and clean. The sum of all the awardee 12. Three times the sum of awardees for cooperation and supervision added to two ti the number of awardees for honesty is 33. If the sum of the number of awardees honesty and supervision is twice the number of awardees for helping others, us matrix method, find the number of awardees of each category. Apart from these val namely, honesty, cooperation and supervision, suggest one more value which management of the colony must include for awards.*	6
40	*10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students and the third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of first and second group is four times that of the third group. Using matrix method, find the number of students in each group. Apart from the values, hard work, honesty and respect for law, vigilance and obedience, suggest one more value, which in your opinion, the school should consider for awards.*	6

41	Using properties of determinants, show the following: $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} - 2abc(a+b+c)^3$	6
42	If a, b, c are positive and unequal, show that the following determinant is negative : $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$	6
43	For what value of k is the following function continuous at $x = 2$? $f(x) = \begin{cases} 2x+1 & ; x < 2 \\ k & ; x = 2 \\ 3x-1 & ; x > 2 \end{cases}$ Differentiate the following with respect to x: $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ Find the equation of tangent to the curve $x = \sin 3t$, $y = \cos 2t$ at $t = \frac{\pi}{4}$	4
44	f $y = \sqrt{x^2+1} - \log\left(\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\right)$, find $\frac{dy}{dx}$.	4
45	Find all points of discontinuity of f, where f is defined as follows : $f(x) = \begin{cases} \lvert x \rvert + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x+2, & x \geq 3 \end{cases}$ ***OR*** Find $\frac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{1/x}$.	4
46	If $\log(x^2+y^2) - 2\tan^{-1}\left(\frac{y}{x}\right)$, then show that $\frac{dy}{dx} - \frac{x+y}{x-y}$. ***OR*** If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, then find $\frac{d^2y}{dx^2}$. Find the equation of the tangent to the curve $y = \sqrt{4x-2}$ which is parallel to the line $4x - 2y + 5 = 0$. ***OR*** Using differentials, find the approximate value of $f(2 \cdot 01)$, where $f(x) = 4x^3 + 5x^2 + 2$.	4
47	. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.	4

48	Show that the function f defined as follows, is continuous at $x = 2$, but not differentiable: $f(x) = \begin{cases} 3x-2, & 0 < x \le 1 \\ 2x^2 - x, & 1 < x \le 2 \\ 5x-4, & x > 2 \end{cases}$ ***OR*** Find $\frac{dy}{dx}$, if $y = \sin^{-1}[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]$.	4
49	If $y = \operatorname{cosec}^{-1} x, x > 1$. then show that $x(x^2-1)\frac{d^2y}{dx^2} + (2x^2-1)\frac{dy}{dx} = 0$	4
50	If $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, show that $\frac{dy}{dx} = \sec x$. Also find the value of $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{4}$. If $y = \cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$, find $\frac{dy}{dx}$.	4
52	If $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a(1 + \sin t)$, find $\frac{d^2y}{dx^2}$.	4
53	Find the value of 'a' for which the function f defined as $f(x) = \begin{cases} a \sin \frac{\pi}{2}(x+1), & x \le 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is continuous at $x = 0$.	4
54	If $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, find $\frac{d^2y}{dx^2}$	4
55	If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$ ***OR*** Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ with respect to x. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t\cos t)$, $0 < t < \frac{\pi}{2}$, find $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.	4

56	Find the relationship between 'a' and 'b' so that the function 'f' defined by: $f(x)=\begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & \text{if } x>3 \end{cases}$ is continuous at $x=3$. **OR** If $x^y = e^{x-y}$, show that $\frac{dy}{dx}=\frac{\log x}{\{\log(xe)\}^2}$. Prove that $y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$ is an increasing function in $\left[0,\frac{\pi}{2}\right]$ **OR** If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area. If $x=\tan\left(\frac{1}{a}\log y\right)$, show that $(1+x^2)\frac{d^2y}{dx^2}+(2x-a)\frac{dy}{dx}=0$	4
57	Prove that : $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]=\sqrt{a^2-x^2}$ **OR** If $y=\log[x+\sqrt{x^2+1}]$, prove that $(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$.	4
58	If $x^m y^n=(x+y)^{m+n}$, prove that $\frac{dy}{dx}=\frac{y}{x}$. If $y=e^{a\cos^{-1}x}$, $-1\le x\le 1$, show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$. **OR** If $x\sqrt{1+y}+y\sqrt{1+x}=0$, $-1<x<1$, $x\ne y$, then prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$.	4
59	If $x^{16}y^9=(x^2+y)^{17}$, prove that $\frac{dy}{dx}=\frac{2y}{x}$.	4
60	Show that the function $f(x)=2x-\|x\|$ is continuous but not differentiable at $x=0$. **OR** Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}x$, when $x\ne 0$.	4
	Complete Integrals from NCERT only till Ex 7.7	